

Statistical analysis of a dynamical multifragmentation path

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A microcanonical multifragmentation model (MMM) is used for investigating whether equilibration *really* occurs in the dynamical evolution of two heavy ion collisions simulated via a stochastic mean field approach (SMF). The standard deviation function between the *dynamically* obtained freeze-out fragment distributions corresponding to the reaction $^{129}\text{Xe}+^{119}\text{Sn}$ at 32 MeV/u and the MMM ones corresponding to a wide range of mass, excitation energy, freeze-out volume and nuclear level density cut-off parameter shows a unique minimum. A distinct statistically equilibrated stage is identified in the dynamical evolution of the system.

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It is known from more than 15 years that violent heavy ion collisions lead to an advanced disassembly of the compound system known under the name of nuclear fragmentation. Good agreements between various observables related to the asymptotically resulted fragments and various models assuming statistical equilibrium [1, 2, 3, 4] lead to the conclusion that a huge part of the available phase space is populated during the fragmentation process. Hence the break-up of a statistically equilibrated nuclear source could be at the origin of fragment production. The source size, its excitation energy and its volume are thus quantities which can only be *indirectly* evaluated by comparisons between experimental data and statistical multifragmentation model predictions via a back-tracing procedure. However, the comparison process is complicated by the presence of several effects, such as pre-equilibrium particle emission, collective radial expansion (see e.g. [5]), Coulomb propagation of the break-up primary fragments, secondary particle emissions. So it is difficult to ascertain whether these quantities really correspond to the source properties obtained at a given time during the fragmentation path. Moreover, there are intrinsic dynamical characteristics such as the freeze-out specific time, directly related to parameters such as the level density cut-off parameter, τ (see Ref. [6]) contributing to the weights of the system statistical ensemble. In the absence of any direct information about the freeze-out this last parameter has to be *employed as a fitting parameter* in a statistical model. In Ref. [6], a very good agreement between statistical fragment distribution predictions and experimental data was obtained *assuming* $\tau = 9$ MeV. However, one has to have *direct* access to the freeze-out events in order to unambiguously decide on the value of such parameters and, if an equilibrated source exists, to find its location in space and time. This task can be achieved using “freeze-out” information from a dynamical model.

This letter investigates whether statistical equilibration occurs in the dynamical path of two heavy ion collision and, if so, which are the corresponding freeze-out parameters. To this aim, the “freeze-out” data of a stochastic mean field (SMF) approach [7] is analyzed via a sharp

microcanonical multifragmentation model (MMM) [4, 6].

We use the stochastic mean-field approach introduced in ref.[8]. According to this theory, the fragmentation process is dominated by the growth of volume (spinodal) and surface instabilities encountered during the expansion phase of the considered excited systems. The dynamical evolution of the system is still described in terms of the one-body distribution function (mean-field description), however this function experiences a stochastic evolution, in response to the action of the fluctuation term. The amplitude of the stochastic term incorporated in the treatment is essentially determined by the degree of thermal agitation present in the system. Hence fluctuations provide the seeds of the formation of fragments, whose characteristics are related to the properties of the most unstable collective modes [9].

In the model [8] fluctuations are implemented only in \mathbf{r} space. Within the assumption of local thermal equilibrium, the kinetic fluctuations typical of a Fermi gas are projected on density fluctuations. Then fluctuations are propagated by the unstable mean-field, leading to the disassembly of the system.

The MMM model concerns the disassembly of a statistically equilibrated nuclear source (A, Z, E, V) (i.e. the source is defined by the parameters: mass number, atomic number, excitation energy and freeze-out volume respectively). Its basic hypothesis is equal probability between all configurations $C : \{A_i, Z_i, \epsilon_i, \mathbf{r}_i, \mathbf{p}_i, i = 1, \dots, N\}$ (the mass number, the atomic number, the excitation energy, the position and the momentum of each fragment i of the configuration C , composed of N fragments) which are subject to standard microcanonical constraints: $\sum_i A_i = A$, $\sum_i Z_i = Z$, $\sum_i \mathbf{p}_i = 0$, $\sum_i \mathbf{r}_i \times \mathbf{p}_i = 0$, E - constant. The fragment level density (entering the statistical weight of a configuration) is of Fermi-gas type adjusted with the cut-off factor $\exp(-\epsilon/\tau)$: $\rho(\epsilon) = \rho_0(\epsilon) \exp(-\epsilon/\tau)$ [6]. The above factor counts for the dramatic decrease of the lifetime of fragment excited states respective to the freeze-out specific time as the excitation energy increases (i.e. earlier freeze-outs should correspond to larger values of τ). The model can work within two freeze-out hypotheses: (1) fragments

are treated as hard spheres placed into a spherical freeze-out recipient and are not allowed to overlap each-other or the recipient wall; (2) fragments may be deformed and a corresponding free-volume expression is approaching the integration over fragment positions [6]. Though more schematic, hypothesis (2) seems more adequate for the present study, so we use it herein. Further a Metropolis-type simulation is employed for determining the average value of any system observable (see Refs. [4] for more details). While the model includes secondary deexcitation, this stage is not needed for this investigation so we will use here only its *primary decay* stage.

Using MMM we investigate whether the “primary events” produced by the stochastic mean-field approach as a result of $^{129}\text{Xe}+^{119}\text{Sn}$ at 32 MeV/u reaction may correspond to the statistical equilibration of the compound system. We consider only very central collisions of the above reaction. According to the dynamical simulations performed in Ref.[7], it is observed that, after the initial collisional shock, the system expands towards low densities entering the unstable region of the nuclear matter phase diagram (after about 100 fm/c from the beginning of the reaction). Then fragments are formed through spinodal decomposition. The *dynamical freeze-out* time is defined as the time when the fragment formation process is over. Hence average fragment multiplicities and distributions do not evolve anymore. For the reaction considered this time is 240 fm/c.

Our aim is to investigate whether fragment distributions are compatible with the statistical phase space occupancy, as predicted by MMM. For washing-up pre-equilibrium effects which should appear in the dynamical simulation, only intermediate mass fragments (IMF) (i.e. fragments with $Z \geq 3$) are selected. Therefore, all comparisons between MMM and stochastic mean-field results are to be restricted to IMF’s. Due to the large Coulomb repulsion among primary fragments it is reasonable to assume that the largest uncertainty in the equilibrated source estimated from the dynamical approach concerns the volume. In other words while *real* equilibration may occur at a different volume, than the one corresponding to the “dynamical” data, we assume that fragment sizes and excitation energies are roughly preserved. Therefore, we will fit the fragment size distributions and their internal excitation energy but *not* the volume. The best fit can be found by minimizing the following error function:

$$\begin{aligned} \mathcal{E} = & \{3[f(\langle A_{\text{bound}} \rangle) + f(\langle Z_{\text{bound}} \rangle)] \\ & + \left[\sum_{N_{\text{IMF}}} f[\langle dN/dN_{\text{IMF}} \rangle] / \sum_{N_{\text{IMF}}} 1 \right] \\ & + f(\langle \epsilon_{\text{IMF}} \rangle) + \sum_{i=1}^3 f(\langle Z_{\text{max}i} \rangle) / 3\} / 9 \end{aligned} \quad (1)$$

where $\langle \cdot \rangle$ stands for average, A_{bound} and Z_{bound} are the bound mass and charge (sum of the mass number and, respectively, atomic number of all IMF’s from a given event), N_{IMF} is the number of IMF’s, ϵ_{IMF} is

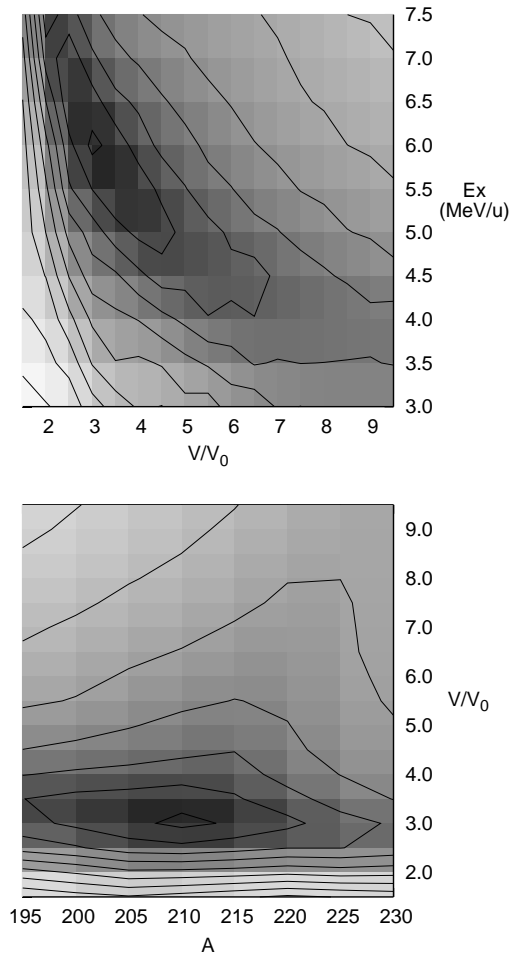


FIG. 1: Contour plots of the error function \mathcal{E} [see eq. (1)]: in the $(V/V_0, E)$ plane corresponding to $A = 210$ (upper panel); in the $(A, V/V_0)$ plane corresponding to $E = 5.7$ MeV/u (lower panel). Darker regions correspond to smaller \mathcal{E} ; units are relative.

the fragment excitation energy per nucleon and $Z_{\text{max}i}$ with $i = 1, 2, 3$ are the largest, second largest and third largest charge from one fragmentation event. Further, $f(x) = |2(x_s - x_d)/(x_s + x_d)|$, where the indexes s and d stand for “statistic” and “dynamic” and $|x|$ is the absolute value of x . One may observe in eq. (1) that the bound mass and charge terms are overweighted with an (arbitrary) factor 3. Of course, this does not influence the significance of the error function, \mathcal{E} , which will vanish for a perfect agreement between the s and d results. On the other side, this will put stronger constraints on source size identification. For finding the best agreement between statistical and dynamical results we variate the MMM parameters A , E , V and τ in *sufficiently* wide ranges thus constructing a four-dimensional grid. The ranges are $A : [195, 230]$, $E : [3, 7.5]$ MeV/u, $V/V_0 : [1.5, 9.5]$, $\tau = 12, 16, \infty$ MeV. The source is considered to have the A/Z ratio of the $^{129}\text{Xe}+^{119}\text{Sn}$ reaction. Considerable computing power was involved for obtaining this result in reasonable time. An absolute minimum of the error function, \mathcal{E} , was found at $A = 210$, $Z = 87$,

TABLE I: Evolution of \mathcal{E}_{\min} (see text) with τ .

τ	12	16	∞
\mathcal{E}_{\min}	0.15	0.11	0.06

$V/V_0 = 3.4$, $E = 5.7$ MeV/u, $\tau = \infty$. Cuts in \mathcal{E} corresponding to $A = 210$ and $E = 5.7$ MeV/u are represented in the upper part and respectively the lower part of Fig. 1. The evolution of \mathcal{E}_{\min} (i.e. the minimum \mathcal{E} corresponding to a given τ) with τ is represented in Table I. From the above mentioned figure one can see that the global minimum is clearly determined and there are no secondary minima.

The statistical source corresponding to the minimum value of \mathcal{E} yields the following results: $\langle A_{\text{bound}} \rangle = 199.03$, $\langle Z_{\text{bound}} \rangle = 83.74$, $\langle \epsilon_{IMF} \rangle = 4.21$ MeV/u, $\langle Z_{\text{max}1} \rangle = 42.34$, $\langle Z_{\text{max}2} \rangle = 24.35$, $\langle Z_{\text{max}3} \rangle = 11.4$. These are to be compared with the corresponding *dynamical* results: $\langle A_{\text{bound}} \rangle = 199$, $\langle Z_{\text{bound}} \rangle = 84$, $\langle \epsilon_{IMF} \rangle = 4.3$ MeV/u, $\langle Z_{\text{max}1} \rangle = 41.95$, $\langle Z_{\text{max}2} \rangle = 22.5$, $\langle Z_{\text{max}3} \rangle = 13.3$. Note the excellent agreement for all considered observables, proving the very good quality of the fit. That means that the fragment size related features obtained dynamically have been reproduced by the primary decay stage of the MMM model. This can be seen in Fig. 3, as well, where the *dynamical* Z and N_{IMF} distributions fit perfectly the MMM ones. Due to the fact that the fragment size distributions and fragment excitations were used for constructing \mathcal{E} these results are natural, given the *small* value of the obtained minimum. At this stage the question still remains: *even if fragment size distributions and excitation energies of the fragments are very well reproduced, do the dynamically formed fragments come from an equilibrated source with freeze-out volume $V = 3.4V_0$, as predicted by MMM?*

In the present work the freeze-out volume V is the volume of the smallest sphere which *totally* includes all fragments. We denote by \tilde{V} the volume of the smallest sphere which *totally* includes all fragments *and* has the center located in the center of mass of the system. Obviously, $\tilde{V} > V$. The “dynamical” events have $\langle (\tilde{V}/V_0)_{IMF} \rangle = 9.08$; the statistical ones have $\langle (\tilde{V}/V_0)_{IMF} \rangle = 4.93$ (the *IMF* index indicates that we refer to the volume occupied by IMF fragments). This means that equilibration, as predicted by MMM, may have occurred at an earlier time compared to the considered dynamical events, i.e. pre-fragments already appear at an earlier time and they are actually equilibrated inside a smaller volume.

One can simply test this hypothesis: one just has to propagate the fragments in their mutual Coulomb field from the freeze-out positioning as generated by MMM up to $(\tilde{V}/V_0)_{IMF} = 9.08$ (i.e. the value corresponding to the “dynamical” events) and then compare “dynamical” and “statistical” fragment kinetic energies and positions. However, in performing such a comparison, one has to

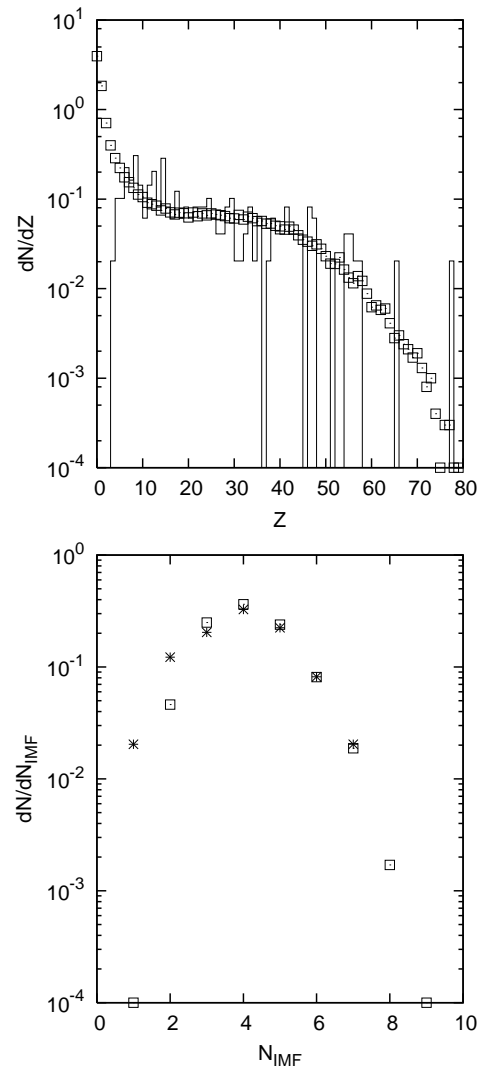


FIG. 2: Dynamical charge (upper panel) and number of IMF distributions (lower panel) in comparisons with the statistical ones. Statistical results are represented by open squares; histogram (upper panel) and stars (lower panel) correspond to the dynamical ones.

guess the initial fragment velocities, that could have a collective component. Flow is easily accountable for in MMM [6]. The best reproduction of the dynamical results is obtained for a flow energy equal to zero. (Note that this result is in agreement with the dynamical simulations where the initial expansion energy present at early times is reduced by the strong interaction between pre-fragments and finally flow becomes close to zero.) The time of the Coulomb propagation is around 100 fm/c. The comparison is presented in Fig. 4: the fragment average kinetic energies versus charge is represented in the upper panel; the radial distribution of the fragment with largest, second largest and third largest charge is represented in the lower panel. An excellent agreement is observed between “dynamical” and “statistical” data for both observables indicating the physical consistency of the scenario used in the present *statistical* analysis.

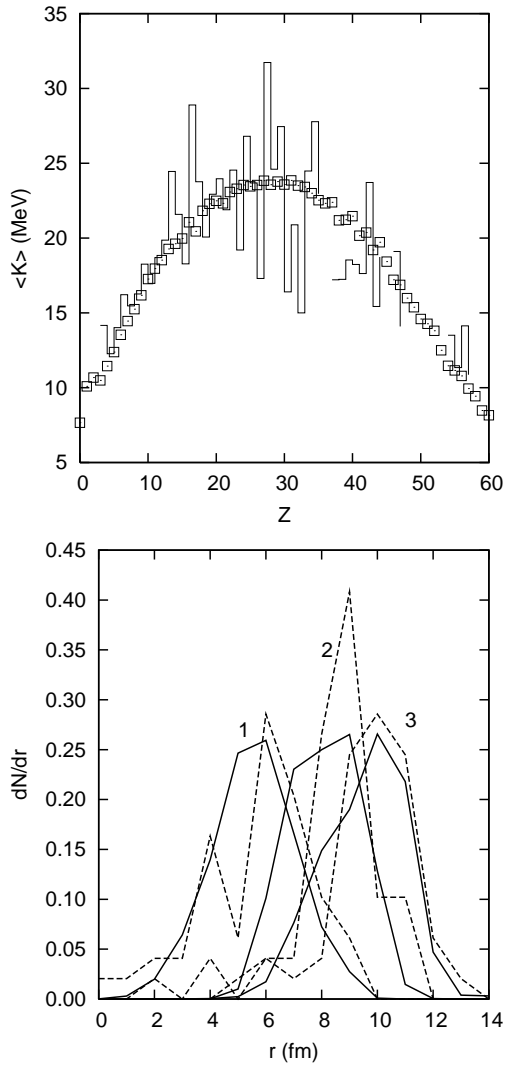


FIG. 3: Upper panel: “statistical” (open squares) and “dynamical” (histogram) fragment average kinetic energy versus charge. Lower panel: “statistical” (solid lines) and “dynamical” (dashed lines) radial distribution of fragments with largest (peak “1”), second largest (peak “2”) and third largest (peak “3”) charge in one fragmentation event. The plot corresponds to $\langle (\tilde{V}/V_0)_{IMF} \rangle = 9.08$.

It means that the dynamically generated events may really originate from a statistically equilibrated source with $V = 3.4V_0$.

Some considerations are now in order. All fragment properties at the *dynamical freeze-out* appear to be almost univocally linked to a statistical emission from a more compact equilibrated source at an earlier time-step, *equilibrium freeze-out*, that eventually expands on Coulomb trajectories. This time ($t \simeq 140 - 150 fm/c$)

can be interpreted as when the dynamical evolution of the system reaches a stage of global equilibrium.

The *equilibrium freeze-out* time range appears just intermediate between the beginning of the spinodal decomposition and the final fragment configuration. At that time the leading unstable modes are well established and some pre-fragments, in strong interaction with the rest of the system, can be recognized. In this situation a statistical fragmentation picture appears adequate, apart some minor effects, as the few “equal fragment size” events, relics of the space structure of the primordial spinodal instability [9]. However, due to the expansion of the system, the *dynamical freeze-out* configuration, where fragments are well formed and separated, corresponds to a larger volume with respect to this *equilibrium freeze-out*.

Since in dynamical simulations fragment formation is coupled with monopole oscillations of the source, so the system may recontract in some events or evolve towards more dilute configurations in some other cases [7] it is obvious that we have to deal with the fluctuation of the freeze-out volume from event to event. On the other hand, in MMM calculations while V/V_0 is fixed, \tilde{V}/V_0 is fluctuating. Whether or not the fluctuations of \tilde{V}/V_0 are sufficient to mimic the “dynamical” ones or one should explicitly include fluctuations of V/V_0 it remains an open question. Since larger fluctuations are reflected into Coulomb effects it would be interesting for future studies to compare the kinetic energy distributions around the average of Fig.4 (top) for each charge.

From the nuclear thermodynamics point of view an important conclusion is that one deals with *small* freeze-out volumes ($3.4 V_0$) and highly excited primary fragments (around 4.3 MeV/u), i.e. *hot fragmentation* at *small* freeze-out volumes. As it is well known, volume is a key variable for locating the system in the phase diagram of the nuclear matter and a lot of work aiming to identify this quantity is in progress [11].

Summarizing, along the fragmentation path, as described by a dynamical model, a huge part of the available phase space is filled. Statistical equilibration occurs in a pre-fragment stage of the system. To our knowledge, this is the first time when in dynamical paths of violent heavy ion collisions a stage of statistical equilibrium of the compound system is proved.

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